MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1 2022/2023 (TERM ID 2210)

EEL 1176 – FIELD THEORY

(All Sections / Groups)

15 FEBRUARY 2023
2.30 p.m. - 4.30 p.m.
(2 hours)

INSTRUCTIONS TO STUDENTS:

1. This question paper consists of 8 pages including the cover page and the Appendix.
2. Answer **ALL** questions. The distribution of the marks for each question is given.
3. Please print all your answers in the Answer Booklet provided.

Question 1

(a) Three uniform vector fields are given by $\vec{A} = 2\hat{x} - \hat{z}$, $\vec{B} = 2\hat{x} - \hat{y} + 2\hat{z}$ and $\vec{C} = 2\hat{x} - 3\hat{y} + \hat{z}$. Determine the value of the following:

(i) $(\vec{A} + \vec{B}) \times (\vec{A} - \vec{B})$. [2 marks]

(ii) $\vec{B} \bullet (\vec{C} \times \vec{A})$. [2 marks]

(b) Given a Cartesian coordinate of P = (3, −4, 2) and a spherical coordinate of Q = (10, π /2, π /3), determine the following:

(i) The Cartesian coordinates of Q. [2 marks]

(ii) The distance between P and Q. [2 marks]

(c) A vector field $\vec{B} = 2xz\hat{x} + (x+2)\hat{y} + y(z^2 - 3)\hat{z}$ exists over all sides (**i** to **vi**) of a cubical box as shown in Figure Q1(c). Verify the Gauss's divergence theorem by evaluating:

(i) $\oint \vec{B} \bullet d\vec{S}$ [12 marks]

(ii) $\oint \nabla \bullet \vec{B} dV$ [5 marks]

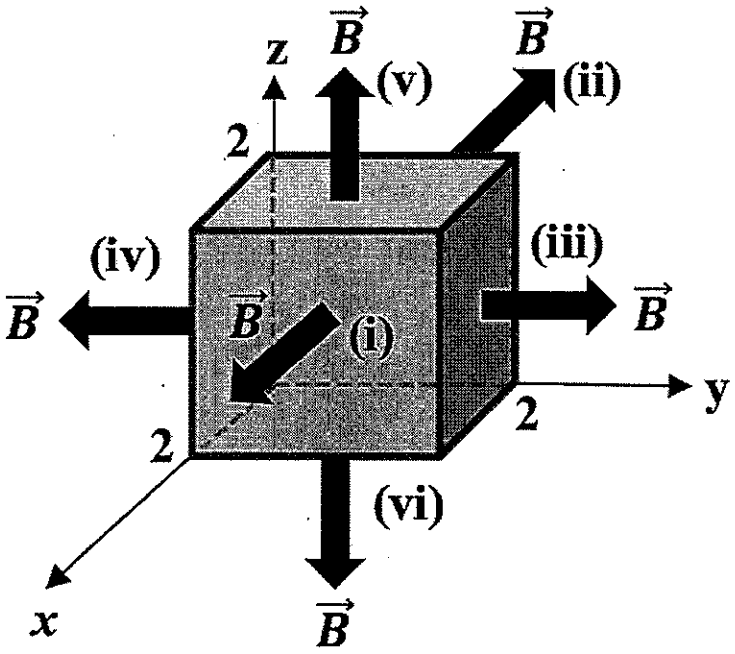

Figure Q1(c)

Continued .....

Question 2

(a) Figure Q2(a) depicts three charges $Q_1 = 4.0nC$, $Q_2 = 4.0nC$, and $Q_3 = 1.0nC$ located at (0, -3, 0) , (0, 3, 0) and (4, 0, 0) respectively. Note that all distances are in meter. Compute the following

(i) The electrical force (F_{13}) on charge Q3 due to charge Q1. [4 marks]

(ii) The electrical force (F_{23}) on charge Q3 due to charge Q2. [4 marks]

(iii) The total force acting on charge Q_3 due to charges Q1 and Q2. [2 marks]

(iv) The electric field intensity on charge Q_3. [2 marks]

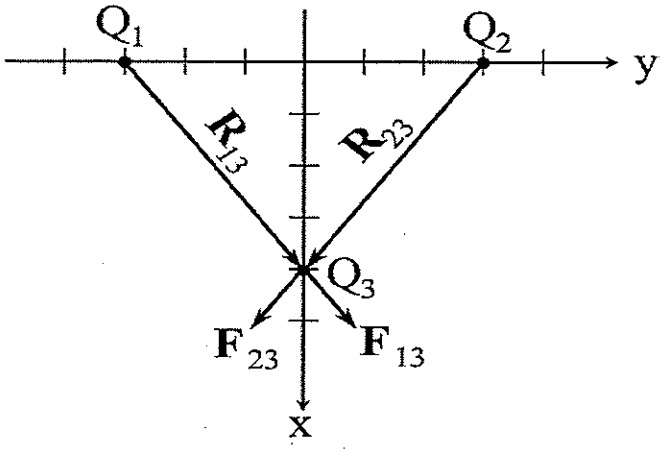

Figure Q2(a)

(b) Given the volume charge density $\rho_v = 1nC/m^3$, find the total charge contained in a cylindrical volume defined by $1 \leq r \leq 5$, $0° \leq \phi \leq 90°$, $0 \leq z \leq 5$.

[4 marks]

(c) A voltage of 10V is applied across a pair of $5 \times 10^{-4} m^2$ parallel plates separated by 2×10^{-3} m dielectric layer with dielectric constant $\varepsilon_r = 5.4$. Compute the following:

(i) The capacitance. [3 marks]

(ii) The magnitude of the charge stored on each plate. [3 marks]

(iii) The electrostatic potential energy. [3 marks]

Continued .....

Question 3

(a) Explain briefly THREE (3) differences of Ampere's Circuital Law and Gauss's Law in Electromagnetism.

[6 marks]

(b) Apply the Ampere's Circuital Law to determine the net current flow for the currents and closed paths enclosed by $\mathbf{L_1}$, $\mathbf{L_2}$, $\mathbf{L_3}$, and $\mathbf{L_4}$ in Figure Q3(b).

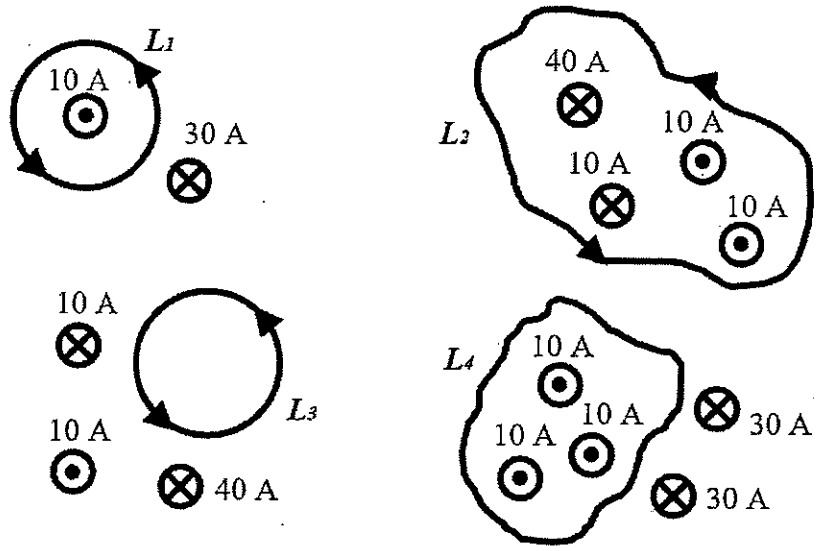

Figure Q3(b)

[4 marks]

(c) A primary long solenoid with $N_1 = 500$, $r_1 = 1$ cm, and $l_1 = 40$ cm is positioned concentrically within a second coil of $N_2 = 1000$, $r_2 = 2$ cm and $l_2 = 40$ cm, where N, r and l are the number of turns, radius and length of the solenoid, respectively. Assuming that both solenoids are filled with air.

(i) If the secondary coil carries a current I_2, determine the self-inductance of the secondary coil L_{22}.

[5 marks]

(ii) If all the flux in the primary coil is linked with the secondary coil, determine the mutual inductance M_{12}.

[5 marks]

(d) A charge of 2.5×10^{-19} C is moving with a speed of 1.5×10^{6} m/s through a magnetic field. The magnetic flux density is 4 T. The charge experiences a magnetic force of magnitude 2.2×10^{-13} N. Determine the angle between the magnetic field and the proton's velocity.

[5 marks]

Continued .....

Question 4

(a) List down susceptibility, χ_m, relative permeability, μ_r and permanent magnetic for the following:

(i) Diamagnetism [3 marks]

(ii) Paramagnetism [3 marks]

(iii) Ferromegnetism [3 marks]

(b) A mild steel ring with a mean radius of 20 cm and cross-sectional area of 2.5 cm^2 is wound with 2000 turns of copper wire. It is given that relative permeability of mild steel is 600. If the copper wire carries a current of 5 A and assume the fringing effect is negligible, evaluate the magnetic flux flow in the iron.

[7 marks]

(c) The magnetic circuit of Figure Q4(c) has current 10 A in the coil of 2000 turns. Assume that all branches have the same cross section of 2 cm^2 and that the material of the core is iron with $\mu_r = 1500$. Assume no fringing field effect and no flux leakage.

(i) Calculate reluctance, *Rc* for the core. [2 marks]

(ii) Calculate reluctance, *Rg* for the air gap. [3 marks]

(iii) Determine the magnetic flux, Φ through the core and the air gap. [2 marks]

(iv) Find the magnetic flux densities, B at the core and the air gap. [2 marks]

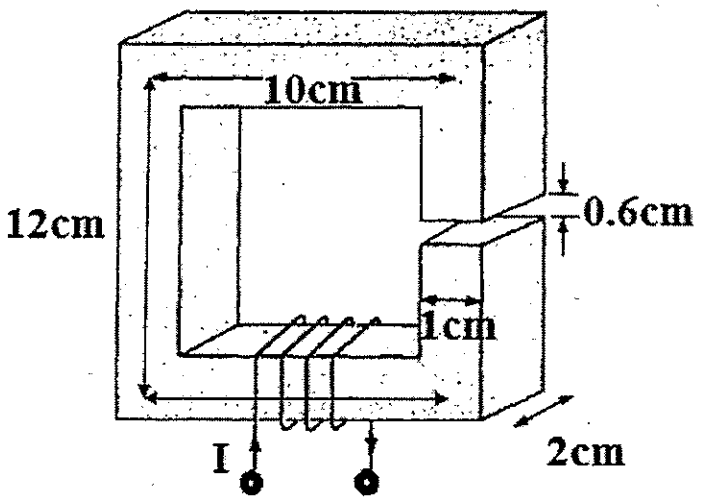

Figure Q4(c)

End of questions.

Appendix

Physical Constants, Vector and Coordinate Transformation Relations

Elementary charge	e	1.60×10^{-19} C
Permittivity constant	ε_0	8.85×10^{-12} F/m
Permeability constant	μ_0	1.26×10^{-6} H/m (or $4\pi \times 10^{-7}$ H/m)

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Differential length	$\hat{x}dx + \hat{y}dy + \hat{z}dz$	$\hat{r}dr + \hat{\phi}rd\phi + \hat{z}dz$	$\hat{R}dR + \hat{\theta}Rd\theta + \hat{\phi}R\sin\theta d\phi$
Differential surface areas	$d\vec{s}_x = \hat{x}dydz$ $d\vec{s}_y = \hat{y}dxdz$ $d\vec{s}_z = \hat{z}dxdy$	$d\vec{s}_r = \hat{r}rd\phi dz$ $d\vec{s}_\phi = \hat{\phi}drdz$ $d\vec{s}_z = \hat{z}rdrd\phi$	$d\vec{s}_R = \hat{R}R^2\sin\theta d\theta d\phi$ $d\vec{s}_\theta = \hat{\theta}R\sin\theta dRd\phi$ $d\vec{s}_\phi = \hat{\phi}RdRd\theta$
Differential volume	$(dx)(dy)(dz)$	$(dr)(rd\phi)(dz)$	$(dR)(Rd\theta)(R\sin\theta d\phi)$

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to Cylindrical	$r = \sqrt{x^2 + y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{r} = \hat{x}\cos\phi + \hat{y}\sin\phi$ $\hat{\phi} = -\hat{x}\sin\phi + \hat{y}\cos\phi$ $\hat{z} = \hat{z}$	$A_r = A_x\cos\phi + A_y\sin\phi$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$ $A_z = A_z$
Cylindrical to Cartesian	$x = r\cos\phi$ $y = r\sin\phi$ $z = z$	$\hat{x} = \hat{r}\cos\phi - \hat{\phi}\sin\phi$ $\hat{y} = \hat{r}\sin\phi + \hat{\phi}\cos\phi$ $\hat{z} = \hat{z}$	$A_x = A_r\cos\phi - A_\phi\sin\phi$ $A_y = A_r\sin\phi + A_\phi\cos\phi$ $A_z = A_z$
Cartesian to Spherical	$R = \sqrt{x^2 + y^2 + z^2}$ $\theta = \tan^{-1}(\sqrt{x^2 + y^2}/z)$ $\phi = \tan^{-1}(y/x)$	$\hat{R} = \hat{x}\sin\theta\cos\phi + \hat{y}\sin\theta\sin\phi + \hat{z}\cos\theta$ $\hat{\theta} = \hat{x}\cos\theta\cos\phi + \hat{y}\cos\theta\sin\phi - \hat{z}\sin\theta$ $\hat{\phi} = -\hat{x}\sin\phi + \hat{y}\cos\phi$	$A_R = A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z\cos\theta$ $A_\theta = A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$
Spherical to Cartesian	$x = R\sin\theta\cos\phi$ $y = R\sin\theta\sin\phi$ $z = R\cos\theta$	$\hat{x} = \hat{R}\sin\theta\cos\phi + \hat{\theta}\cos\theta\cos\phi - \hat{\phi}\sin\phi$ $\hat{y} = \hat{R}\sin\theta\sin\phi + \hat{\theta}\cos\theta\sin\phi + \hat{\phi}\cos\phi$ $\hat{z} = \hat{R}\cos\theta - \hat{\theta}\sin\theta$	$A_x = A_R\sin\theta\cos\phi + A_\theta\cos\theta\cos\phi - A_\phi\sin\phi$ $A_y = A_R\sin\theta\sin\phi + A_\theta\cos\theta\sin\phi + A_\phi\cos\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$
Cylindrical to Spherical	$R = \sqrt{r^2 + z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{R} = \hat{r}\sin\theta + \hat{z}\cos\theta$ $\hat{\theta} = \hat{r}\cos\theta - \hat{z}\sin\theta$ $\hat{\phi} = \hat{\phi}$	$A_R = A_r\sin\theta + A_z\cos\theta$ $A_\theta = A_r\cos\theta - A_z\sin\theta$ $A_\phi = A_\phi$
Spherical to Cylindrical	$r = R\sin\theta$ $\phi = \phi$ $z = R\cos\theta$	$\hat{r} = \hat{R}\sin\theta + \hat{\theta}\cos\theta$ $\hat{\phi} = \hat{\phi}$ $\hat{z} = \hat{R}\cos\theta - \hat{\theta}\sin\theta$	$A_r = A_R\sin\theta + A_\theta\cos\theta$ $A_\phi = A_\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$

Gradient, Divergence, Curl and Laplacian Operators

Cartesian coordinate (x, y, z)

$$\nabla V = \hat{x}\frac{\partial V}{\partial x} + \hat{y}\frac{\partial V}{\partial y} + \hat{z}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \tilde{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \tilde{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \hat{x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{y}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

Cylindrical coordinate (r, ϕ, z)

$$\nabla V = \hat{r}\frac{\partial V}{\partial r} + \hat{\phi}\frac{1}{r}\frac{\partial V}{\partial \phi} + \hat{z}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \tilde{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \tilde{A} = \frac{1}{r}\begin{vmatrix} \hat{r} & \hat{\phi}r & \hat{z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix} = \hat{r}\left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{\phi}\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \hat{z}\frac{1}{r}\left[\frac{\partial}{\partial r}\left(rA_\phi\right) - \frac{\partial A_r}{\partial \phi}\right]$$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

Spherical coordinate (R, θ, ϕ)

$$\nabla V = \hat{R}\frac{\partial V}{\partial R} + \hat{\theta}\frac{1}{R}\frac{\partial V}{\partial \theta} + \hat{\phi}\frac{1}{R\sin\theta}\frac{\partial V}{\partial \phi}$$

$$\nabla \cdot \tilde{A} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 A_R) + \frac{1}{R\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \tilde{A} = \frac{1}{R^2\sin\theta}\begin{vmatrix} \hat{R} & \hat{\theta}R & \hat{\phi}R\sin\theta \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_R & RA_\theta & (R\sin\theta)A_\phi \end{vmatrix}$$

$$= \hat{R}\frac{1}{R\sin\theta}\left[\frac{\partial}{\partial \theta}\left(A_\phi \sin\theta\right) - \frac{\partial A_\theta}{\partial \phi}\right] + \hat{\theta}\frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial \phi} - \frac{\partial}{\partial R}\left(RA_\phi\right)\right] + \hat{\phi}\frac{1}{R}\left[\frac{\partial}{\partial R}\left(RA_\theta\right) - \frac{\partial A_R}{\partial \theta}\right]$$

$$\nabla^2 V = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

Table of Integrals

$$\int \sin^2\theta\, d\theta = \frac{\theta}{2} - \frac{1}{4}\sin 2\theta + \text{constant}$$

$$\int \cos^2\theta\, d\theta = \frac{\theta}{2} + \frac{1}{4}\sin 2\theta + \text{constant}$$

$$\int \sin\theta\cos^2\theta\, d\theta = -\frac{1}{3}\cos^3\theta + \text{constant}$$

$$\int \cos\theta\sin^4\theta\, d\theta = \frac{1}{5}\sin^5\theta + \text{constant}$$

$$\int \sin 2\theta\, d\theta = -\frac{1}{2}\cos 2\theta + \text{constant}$$

$$\int x\sqrt{a^2 - x^2}\, dx = -\frac{1}{3}\left(a^2 - x^2\right)^{3/2} + \text{constant}$$

$$\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\tan^{-1}\left(\frac{x}{\sqrt{a^2 - x^2}}\right) + \text{constant}$$

$$\int \frac{x}{\sqrt{a^2 - x^2}} dx = -\sqrt{a^2 - x^2} + \text{constant}$$

$$\int \frac{dz}{\left(r^2 + z^2\right)^{3/2}} = \frac{z}{r^2\sqrt{r^2 + z^2}} + \text{constant}$$

$$\int \frac{rdr}{\left(r^2 + z^2\right)^{3/2}} = -\frac{1}{\sqrt{r^2 + z^2}} + \text{constant}$$